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Statistical Properties of the Generalized Photon-Added Pair Coherent State

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Abstract We present a class of generalized photon-added pair coherent states (GPAPCS) and analyze some prominent nonclassical properties such as sub-Poissonian distribution and violations of Cauchy-Schwarz inequalities. In addition, we derive that the Wigner function of GPAPCS involves correlation of two two-variable Hermite polynomials and its Husimi function is related to a two-variable Hermite polynomial. Their behaviors varying with the phase space parameters are also graphically discussed. We find that the nonclassical effects of GPAPCS exhibits more with increasing of excitation photon numbers.

Keywords Photon-added pair coherent state · Sub-Poissonian statistics · Wigner function · Husimi function

1 Introduction

Nonclassicality of optical fields has been a topic of great interest, beyond the traditional realm of quantum optics, in research fields of great current interest, such as laser pulsed atoms and molecules [1], Bose-Einstein condensation and atom lasers [2, 3], and quantum information theory [4]. Some single-mode nonclassical states, such as squeezed states [5], photon-added coherent state [6], even and odd coherent stats [7, 8], displaced and squeezed Fock stats [9], differ essentially from the statistics of coherent state [10, 11] which has Poissonian or sub-Poissonian photon distribution and Gaussian quadrature statistics with equal minimal dispersions of both noncorrelated quadratures.

On the other hand, pair coherent states (PCS) [12–14] are regarded as an important type of correlated two-mode states [15, 16], which involve prominent nonclassical properties

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such as sub-Poissonian statistics, correlations in the number fluctuations, squeezing, and violations of Cauchy-Schwarz inequalities. The PCS in the Fock space is defined as

$$|q,\zeta\rangle = C_q \sum_{n=0}^{\infty} \frac{\zeta^n}{\sqrt{(n+q)!n!}} |n+q,n\rangle, \quad \zeta = |\zeta| e^{i\theta}, \tag{1}$$

where q is the photon number difference between two modes and C_q is the normalization constant,

$$C_q = \left(\sum_{n=0}^{\infty} \frac{|\zeta|^{2n}}{(n+q)!n!} \right)^{-1/2} = \left[(i|\zeta|)^{-q} J_q(2i|\zeta|) \right]^{-1/2}, \tag{2}$$

J_q is the ordinary Bessel function. In fact, the PCS $|q,\zeta\rangle$ is the common eigenvector of both the pair-annihilation operator (ab) and the difference in the number operator $Q = a^\dagger a - b^\dagger b$ for the two modes, i.e.,

$$ab|q,\zeta\rangle = \zeta|q,\zeta\rangle, \qquad Q|q,\zeta\rangle = q|q,\zeta\rangle \tag{3}$$

and is related to the coherent state $|z\rangle_a |z\rangle_b$ by

$$|q,\zeta\rangle = C_q \int \frac{d\theta}{2\pi} e^{|\zeta|} \left(\zeta^{1/2} e^{i\theta} \right)^{-q} |\zeta^{1/2} e^{i\theta}\rangle_a |\zeta^{1/2} e^{-i\theta}\rangle_b, \tag{4}$$

which is called its Schmidt decomposition. Recently, [17] suggested for generating vibrational PCS via the motion of a trapped ion in a two-dimensional trap.

In the present paper, we introduce the generalized photon-added pair coherent states (GPAPCS) $|q,\zeta,k,l\rangle$, which are constructed by repeatedly operating the creation operators $a^\dagger$ and $b^\dagger$ on $|q,\zeta\rangle$,

$$\begin{aligned} |q,\zeta,k,l\rangle &= C_q' a^{\dagger k} b^{\dagger l} |q,\zeta\rangle \\ &= A_q \sum_{n=0}^{\infty} D_n \, |n+q+k, n+l\rangle, \end{aligned} \tag{5}$$

where C_q' is introduced for normalization,

$$D_n = \frac{\sqrt{(n+q+k)!\,(n+l)!}\zeta^n}{(n+q)!n!} \tag{6}$$

and

$$A_q = C_q' C_q = \left[\sum_{n=0}^{\infty} \frac{(n+q+k)!(n+l)!|\zeta|^{2n}}{[(n+q)!n!]^2} \right]^{-1/2}. \tag{7}$$

Especially, when $k = l \neq 0$, the GPAPCS becomes the excited PCS in [18], so $|q,\zeta,k,l\rangle$ is named the generalized photon-added pair coherent state. We shall examine sub-Poissonian distribution and violation of Cauchy-Schwarz inequality for $|q,\zeta,k,l\rangle$. Subsequently, we derive the explicit expression of Wigner function for GPAPCS. The result shows that the Wigner function of GPAPCS involves correlation of two two-variable Hermite polynomials, which is not only concise, but also reveals entanglement involved in GPAPCS. Its Husimi function is calculated as well, which is related to a two-variable Hermite polynomial. In addition, we also examine how its Wigner function and the Husimi function varies with the phase space parameters.

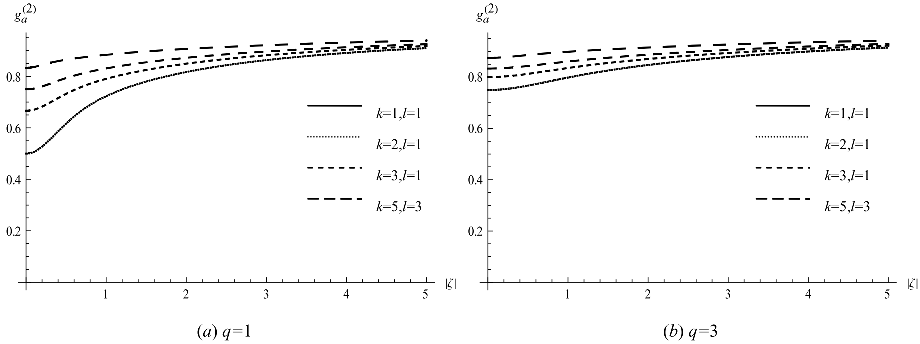


Fig. 1 Second-order correlation function $g_a^{(2)}(\zeta)$ of GPAPCS with $q=1$, $q=3$ for several different k and l

2 Sub-Poissonian Statistics and Violation of Cauchy-Schwarz Inequality

It is well known that sub-Poissonian statistics is characterized by the fact that the variance of the photon number $\langle(\Delta n_j)^2\rangle$ is less than the average photon number $\langle n_j\rangle$. This can be expressed by means of the normalized second-order correlation function [19] for mode j in $|q,\zeta,k,l\rangle$ as follows

$$g_j^{(2)}(\zeta)=\frac{\langle q,\zeta,k,l|n_j(n_j-1)|q,\zeta,k,l\rangle}{\langle q,\zeta,k,l|n_j|q,\zeta,k,l\rangle^2},\qquad j=a,b. \tag{8}$$

By using (5), the mean numbers of photons n_a and n_b are given by

$$\langle q,\zeta,k,l|n_a|q,\zeta,k,l\rangle=A_q^2\sum_{n=0}^{\infty}|D_n|^2(n+q+k), \tag{9}$$

and

$$\langle q,\zeta,k,l|n_b|q,\zeta,k,l\rangle=A_q^2\sum_{n=0}^{\infty}|D_n|^2(n+l). \tag{10}$$

Similarly, we obtain the following expectation values

$$\langle q,\zeta,k,l|n_a(n_a-1)|q,\zeta,k,l\rangle=A_q^2\sum_{n=0}^{\infty}|D_n|^2(n+q+k)(n+q+k-1), \tag{11}$$

and

$$\langle q,\zeta,k,l|n_b(n_b-1)|q,\zeta,k,l\rangle=A_q^2\sum_{n=0}^{\infty}|D_n|^2(n+l)(n+l-1). \tag{12}$$

The second-order correlation function $g_j^{(2)}(\zeta)$ given by (8) for the mode j serves as a measure of the deviation from the Poissonian distribution that corresponds to coherent states with $g_j^{(2)}(\zeta)=1$. If $g_j^{(2)}(\zeta)<1(>1)$, the distribution is called sub(super)-Poissonian. If $g_j^{(2)}(\zeta)=2$, the distribution is named thermal and when $g_j^{(2)}(\zeta)>2$, it is called superthermal.

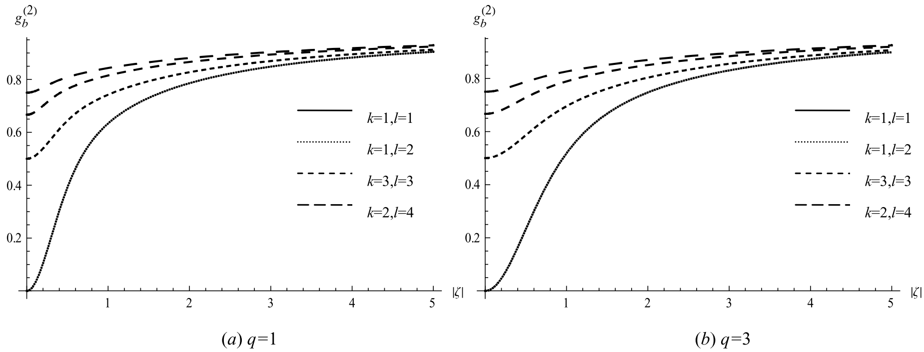
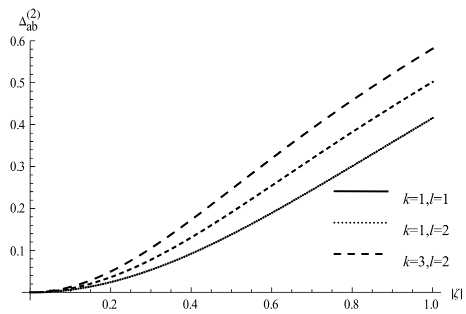


Fig. 2 Second-order correlation function $g_b^{(2)}(\zeta)$ of the GPAPCS with $q = 1$, $q = 3$ for several different k and l

Fig. 3 Cross correlation function $\Delta_{cross}(\zeta)$ of GPAPCS against $|\zeta|$ with $q = 3$ for several different k and l



To reveal the physical content of $|q, \zeta, k, l\rangle$, we numerically present the function $g_i^{(2)}(\zeta)$ against $|\zeta|$ for different k, l and q. In Fig. 1, as $q = 1$ or $q = 3$, the function $g_a^{(2)}(\zeta)$ is full sub-Poissonian distribution. From Fig. 1(a) and (b), it is seen clearly that the values of $g_a^{(2)}(\zeta)$ become larger with increasing the number k (l) of photon-added in mode a (b). Especially, $g_a^{(2)}(\zeta) \to 1$ for the large value of $|\zeta|$. In Fig. 2, $g_b^{(2)}(\zeta)$ is depicted with $q = 1$ and $q = 3$ for several different k and l, whose behavior is similar to the function $g_a^{(2)}(\zeta)$. So the function $g_a^{(2)}(\zeta)$ ($g_b^{(2)}(\zeta)$) is affected by parameters k and l significantly.

Moreover, the cross correlation between two modes is defined by

$$\Delta_{cross}(\zeta) = \langle n_a n_b\rangle - \langle n_a\rangle\langle n_b\rangle. \tag{13}$$

If $\Delta_{cross}(\zeta)$ is a positive quantity, this means that the modes a and b are correlated, while anti-correlation amongst the modes occurs when $\Delta_{cross}(\zeta)$ is negative values. For $|q, \zeta, k, l\rangle$, we have

$$\langle q, \zeta, k, l|n_a n_b|q, \zeta, k, l\rangle = A_q^2 \sum_{n=0}^{\infty} |D_n|^2 (n + q + k)(n + l). \tag{14}$$

In Fig. 3, the cross correlation function $\Delta_{cross}(\zeta)$ of $|q, \zeta, k, l\rangle$ is plotted against $|\zeta|$ with $q = 3$ for the different k and l, respectively. On this figure, $\Delta_{cross}(\zeta)$ is positive and the nonclassical behavior is demonstrated by increase of k and l.

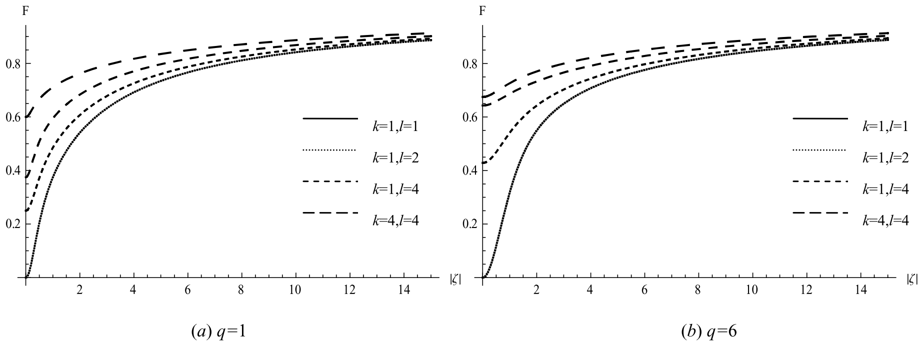


Fig. 4 Function $F_{ab}(\zeta)$ of GPAPCS against $|\zeta|$ with $q = 1$, $q = 6$ for several different k and l

Next, we examine Cauchy-Schwarz inequality in $|q, \zeta, k, l\rangle$, which is determined by

$$F_{ab}(\zeta) = \frac{\langle q, \zeta, k, l|n_a(n_a - 1)|q, \zeta, k, l\rangle\langle q, \zeta, k, l|n_b(n_b - 1)|q, \zeta, k, l\rangle}{(\langle q, \zeta, k, l|n_a n_b|q, \zeta, k, l\rangle)^2}. \tag{15}$$

If the function $F_{ab}(\zeta)$ is less than unity, Cauchy-Schwarz inequality is violated. The Cauchy-Schwarz inequality for $|q, \zeta, k, l\rangle$ is clearly seen in Fig. 4. The function $F_{ab}(\zeta) < 1$, so the GPAPCS $|q, \zeta, k, l\rangle$ full violates this inequality. As the parameter $|\zeta|$ increases the function $F_{ab}(\zeta)$ reaches unity.

3 Wigner Function of GPAPCS

In studying quantum mechanics and quantum statistics, the presence of negativity in the Wigner function of an optical field is a signature of its nonclassicality [20–22]. In this section, we want to evaluate the Wigner function for GPAPCS.

For a single-mode system, the Wigner operator $\Delta_w(\lambda)$ in the coherent state representation is given by [23, 24]

$$\begin{aligned}\Delta_w(\lambda) &= e^{2|\lambda|^2}\int \frac{d^2z}{\pi^2}|z\rangle\langle -z|\exp\left[-2(z\lambda^* - z^*\lambda)\right] \\ &= \frac{1}{\pi}:\ \exp\left[-2(a^\dagger - \lambda^*)(a - \lambda)\right]:\end{aligned} \tag{16}$$

where

$$|z\rangle = \exp\left(-\frac{|z|^2}{2} + za^\dagger\right)|0\rangle = e^{-\frac{|z|^2}{2}}\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}|n\rangle \tag{17}$$

is the coherent state [10, 11], $:\ :$ denotes normal ordering for $(a^\dagger, a)$ and $\lambda \equiv \frac{1}{\sqrt{2}}(x + ip)$ with x, p being the eigenvalues of the operators X, P, respectively. So the corresponding Wigner function for $|q, \zeta, k, l\rangle$, is defined by

$$W(\alpha, \beta) = \langle q, \zeta, k, l|\Delta_w(\alpha)\Delta_w(\beta)|q, \zeta, k, l\rangle, \tag{18}$$

where

$$\alpha \equiv \frac{1}{\sqrt{2}}(x_1 + ip_1), \qquad \beta \equiv \frac{1}{\sqrt{2}}(x_2 + ip_2). \tag{19}$$

In order to obtain Wigner function of $|q,\zeta,k,l\rangle$, we first calculate the expression $\langle m|\Delta_w(\alpha)|n\rangle$ where $|n\rangle$ and $|m\rangle$ is the number states. Using (16) and noticing $\langle z|n\rangle = \frac{1}{\sqrt{n!}} z^{*n} e^{-\frac{1}{2}|z|^2}$, we have

$$\begin{aligned}
\langle m|\Delta_w(\alpha)|n\rangle &= e^{2|\alpha|^2} \int \frac{d^2z}{\pi^2} \langle m|z\rangle\langle -z|n\rangle \exp\left[-2\left(z\alpha^* - z^*\alpha\right)\right] \\
&= e^{2|\alpha|^2} \frac{(-1)^n}{\sqrt{m!n!}} \int \frac{d^2z}{\pi^2} z^m \left(z^*\right)^n \exp\left[-|z|^2 + \left(-2\alpha^*\right) z - (-2\alpha)\, z^*\right] \\
&= e^{-2|\alpha|^2} \frac{(-1)^{n+m}}{\pi\sqrt{m!n!}} H_{m,n}\left(-2\alpha, -2\alpha^*\right) \\
&= \frac{e^{-2|\alpha|^2}}{\pi\sqrt{m!n!}} H_{m,n}\left(2\alpha, 2\alpha^*\right),
\end{aligned} \tag{20}$$

where in the above derivation we have used the integration expression of two-variable Hermite polynomial $H_{m,n}(\xi,\eta)$,

$$H_{m,n}(\xi,\eta) = (-1)^n e^{\xi\eta} \int \frac{d^2z}{\pi} z^n z^{*m} \exp\left[-|z|^2 + \xi z - \eta z^*\right], \tag{21}$$

whose definition is [25]

$$H_{m,n}(\xi,\eta) = \sum_{l=0}^{\min(m,n)} \frac{(-1)^l\, m!n!\,\xi^{m-l}\eta^{n-l}}{l!\,(n-l)!\,(m-l)!} = H_{n,m}(\eta,\xi). \tag{22}$$

Thus, by substituting (5) into (18) and using (6) and (20), the explicit expression of Wigner function for GPAPCS is

$$\begin{aligned}
W(\alpha,\beta) &= \langle q,\zeta,k,l|\Delta_w(\alpha)\,\Delta_w(\beta)\,|q,\zeta,k,l\rangle \\
&= A_q \sum_{m=0}^{\infty} D_m^* \langle m+q+k, m+l|\, \Delta_w(\alpha)\,\Delta_w(\beta)\, A_q \sum_{n=0}^{\infty} D_n\, |n+q+k, n+l\rangle \\
&= A_q^2 \sum_{m,n=0}^{\infty} D_m^* D_n \langle m+q+k|\, \Delta_w(\alpha)\,|n+q+k\rangle\, \langle m+l|\, \Delta_w(\beta)\,|n+l\rangle \\
&= \frac{A_q^2 e^{-2|\alpha|-2|\beta|^2}}{\pi^2} \sum_{m,n=0}^{\infty} \frac{(\zeta^*)^m \zeta^n H_{m+q+k,n+q+k}(2\alpha,2\alpha^*)\, H_{m+l,n+l}(2\beta,2\beta^*)}{(m+q)!(n+q)!m!n!}.
\end{aligned} \tag{23}$$

For the entangled system, the Wigner function embodies entanglement through variable

$$2\alpha \to \gamma + \sigma,\, 2\beta^* \to \gamma - \sigma, \tag{24}$$

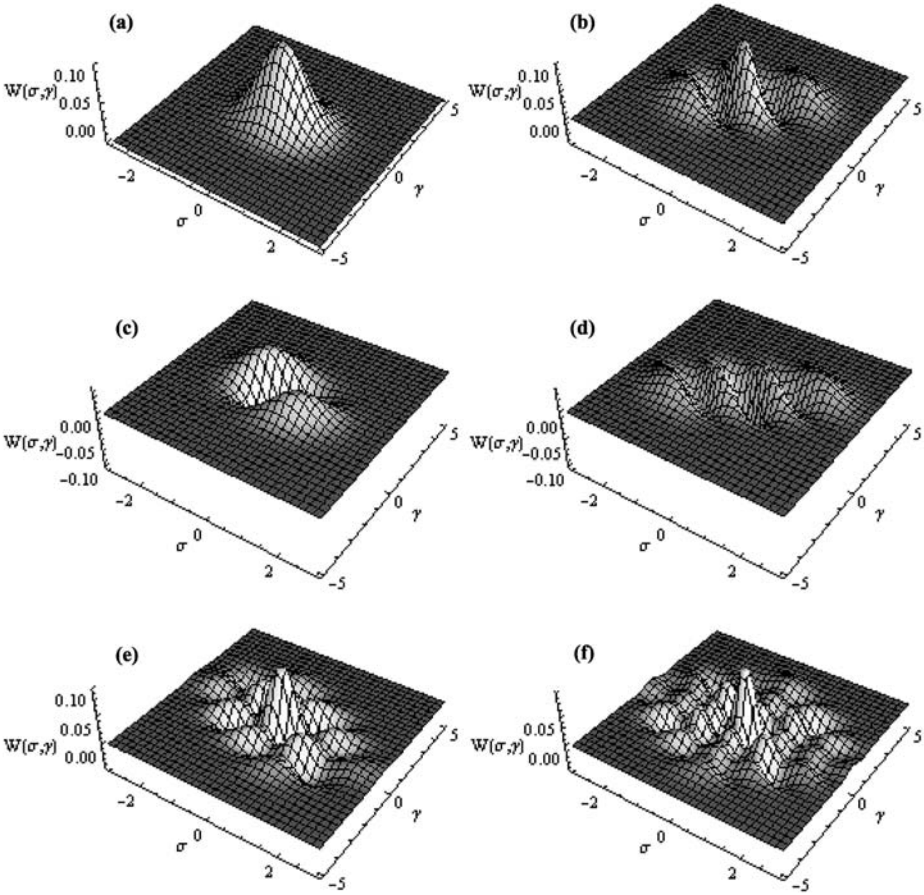


Fig. 5 Wigner functions $W(\sigma,\gamma)$ of the GPAPCS against σ and γ (fixed at $\mathrm{Im}\,\sigma = \mathrm{Im}\,\gamma = 0$) for $|\zeta| = 0.1$: (**a**) $q = k = l = 0$, (**b**) $q = l = 0$ and $k = 2$, (**c**) $q = k = 0$ and $l = 1$, (**d**) $q = 1$, $k = 2$ and $l = 0$, (**e**) $q = 1$, $k = 0$ and $l = 3$, (**f**) $q = 1$, $k = 2$ and $l = 3$, respectively

so

$$W(\alpha,\beta) \to W(\sigma,\gamma). \tag{25}$$

Equation (23) involves correlation of two two-variable Hermite polynomials. This in turn exhibits that the GPAPCS is also an entangled state. In particular, $k = l = 0$, the Wigner function $W(\sigma,\gamma)$ of GPAPCS in (23) reduces to that of PCS, which is equal to (22) and the corresponding graphics is plotted in [26].

Now we discuss changes in the Wigner function $W(\sigma,\gamma)$ fixed at $\mathrm{Im}\,\sigma = \mathrm{Im}\,\gamma = 0$ as we vary the parameters q, k and l. When $q = k = l = 0$ and $|\zeta| = 0.1$ in Fig. 5(a), the function $W(\sigma,\gamma)$ displays a single peak and has Gaussian shape. The number of downward peak is equal to the value of q while the number of upward peak is $q + 1$ [26]. From Fig. 5(a)–(e), for the fixed value of q, the increasing number of upward peaks equals to $k + 1$ $(l + 1)$, but the increasing downward or upward peaks of $W(\sigma,\gamma)$ exist in the orthogonal orientation. In Fig. 5(f), we also see that GPAPCS exhibits remarkably quantum interference effects. It is clear from Fig. 5 that the Wigner function has negative parts with increasing the value of q, k and l, therefore nonclassical effects can be remarkably established.

4 Husimi Function of GPAPCS

As many authors have pointed out that the Wigner function is not is a probability distribution since it may takes on both positive and negative values. To overcome this shortcomings, the so-called Husimi distribution function is introduced [27], which is defined in a manner that guarantees it to be non-negative and gives it a probability interpretation. Its definition is smoothing out the Wigner function by averaging over a "coarse graining" function. The Husimi operator $\Delta_h(\sigma,\gamma)$ which corresponds to Husimi function is defined as [28]

$$\Delta_h(\sigma,\gamma,\kappa)=4\int d^2\sigma' d^2\gamma' \Delta_w(\sigma',\gamma')\exp\left(-\kappa\left|\sigma'-\sigma\right|^2-\frac{\left|\gamma'-\gamma\right|^2}{\kappa}\right), \tag{26}$$

where κ is the Gaussian spatial width parameter and $\Delta_w(\sigma',\gamma')\equiv\Delta_w(\alpha')\Delta_w(\beta')$ is two-mode Wigner operator with $\gamma'\equiv\alpha'+\beta'^*$, $\sigma'\equiv\alpha'-\beta'^*$. Using (16) and performing the integration in (26), we obtain normally ordered form of the Husimi operator $\Delta_h(\sigma,\gamma)$

$$\begin{aligned}\Delta_h(\sigma,\gamma,\kappa)=\frac{4\kappa}{(1+\kappa)^2}:\ \exp\Big[&-\frac{1}{1+\kappa}(a+b^\dagger-\gamma)(a^\dagger+b-\gamma^*)\\&-\frac{\kappa}{1+\kappa}(\sigma-a+b^\dagger)(\sigma^*-a^\dagger+b)\Big]:.\end{aligned} \tag{27}$$

In [29–31], the Husimi operator $\Delta_h(\sigma,\gamma)$ may be expressed as

$$\Delta_h(\sigma,\gamma,\kappa)=|\sigma,\gamma\rangle_{\kappa\kappa}\langle\sigma,\gamma|, \tag{28}$$

where $|\sigma,\gamma\rangle_\kappa$ is a kind two-mode squeezed coherent state

$$|\sigma,\gamma\rangle_\kappa=C_\kappa(\sigma,\gamma)\exp\left(\frac{\kappa\sigma+\gamma}{1+\kappa}a^\dagger+\frac{\gamma^*-\kappa\sigma^*}{1+\kappa}b^\dagger+\frac{\kappa-1}{1+\kappa}a^\dagger b^\dagger\right)|00\rangle \tag{29}$$

and

$$C_\kappa(\sigma,\gamma)=\frac{2\sqrt{\kappa}}{1+\kappa}\exp\left[-\frac{1}{2(1+\kappa)}\left(\kappa|\sigma|^2+|\gamma|^2\right)\right]. \tag{30}$$

When $\kappa=1$,

$$|\sigma,\gamma\rangle_1=\exp\left[-\frac{1}{4}\left(|\sigma|^2+|\gamma|^2\right)+\frac{\sigma+\gamma}{2}a^\dagger+\frac{\gamma^*-\sigma^*}{2}b^\dagger\right]|00\rangle \tag{31}$$

is a two-mode canonical coherent state. Then, the formula to calculate Husimi distribution function of a pure state system $|\psi\rangle$ may become simpler,

$$\mathcal{H}(\sigma,\gamma,\kappa)=\mathrm{Tr}\left[|\psi\rangle\langle\psi|\,\Delta_h(\sigma,\gamma,\kappa)\right]=|_\kappa\langle\sigma,\gamma|\,\psi\rangle|^2. \tag{32}$$

To calculate the Husimi function of GPAPCS, using (5), (17) and (29), we obtain the overlap relation

$$_\kappa\langle\sigma,\gamma|\,q,\zeta,k,l\rangle=A_q\sum_{n=0}^{\infty}C_\kappa(\sigma,\gamma)\frac{\zeta^n}{(n+q)!n!}\int\frac{d^2z_1d^2z_2}{\pi^2}z_1^{*n+q+k}z_2^{*n+l}$$

$$\times \exp\left(\frac{\kappa\sigma^* + \gamma^*}{1+\kappa} z_1 + \frac{\gamma - \kappa\sigma}{1+\kappa} z_2 + \frac{\kappa - 1}{1+\kappa} z_1 z_2 - |z_1|^2 - |z_2|^2\right). \tag{33}$$

Further by considering another expression of two-variable Hermite polynomial [25]

$$H_{m,n}(\xi, \eta) = \frac{\partial^{m+n}}{\partial t^m \partial t'^n} \exp\left(-tt' + t\xi + t'\eta\right)\Big|_{t=t'=0}, \tag{34}$$

and utilizing the following integral formula

$$\int \frac{d^2 z}{\pi} \exp\left(\varsigma |z|^2 + \zeta z + \xi z^* + f z^2 + g z^{*2}\right)$$
$$= \frac{1}{\sqrt{\varsigma^2 - 4fg}} \exp\left(\frac{-\varsigma\zeta\xi + \zeta^2 f + \xi^2 g}{\varsigma^2 - 4fg}\right) \tag{35}$$

with

$$\mathrm{Re}\,(\varsigma \pm f \pm g) < 0, \qquad \mathrm{Re}\left(\frac{\varsigma^2 - 4fg}{\varsigma \pm f \pm g}\right) < 0, \tag{36}$$

the integration of (33) is simplified as

$$\int \frac{d^2 z_1 d^2 z_2}{\pi^2} z_1^{*n+q+k} z_2^{*n+l} \exp\left(\frac{\kappa\sigma^* + \gamma^*}{1+\kappa} z_1 + \frac{\gamma - \kappa\sigma}{1+\kappa} z_2 + \frac{\kappa - 1}{1+\kappa} z_1 z_2 - |z_1|^2 - |z_2|^2\right)$$
$$= \frac{\partial^{n+l}}{\partial \lambda^{n+l}} \frac{\partial^{n+q+k}}{\partial \mu^{n+q+k}} \int \frac{d^2 z_1}{\pi} \exp\left(-|z_1|^2 + \left(\frac{\kappa\sigma^* + \gamma^*}{1+\kappa} + \frac{\kappa - 1}{1+\kappa}\lambda\right) z_1\right.$$
$$\left. + \mu z_1^* + \frac{\gamma - \kappa\sigma}{1+\kappa}\lambda\right)\Big|_{\lambda=\mu=0}$$
$$= \frac{\partial^{n+l}}{\partial \lambda^{n+l}} \frac{\partial^{n+q+k}}{\partial \mu^{n+q+k}} \exp(L\lambda\mu + M\mu + N\lambda)|_{\lambda=\mu=0},$$
$$= \left(i\sqrt{L}\right)^{n+l+n+q+k} H_{n+l,n+q+k}\left(\frac{N}{i\sqrt{L}}, \frac{M}{i\sqrt{L}}\right), \tag{37}$$

where

$$M \equiv \frac{\kappa\sigma^* + \gamma^*}{1+\kappa}, \qquad N \equiv \frac{\gamma - \kappa\sigma}{1+\kappa}, \qquad L \equiv \frac{\kappa - 1}{1+\kappa}. \tag{38}$$

Therefore, using (32) and (33) and (37), the Husimi function of GPAPCS is finally rewritten as

$$\mathcal{H}(\sigma, \gamma, \kappa) = A_q^2 \left| \sum_{n=0}^{\infty} \frac{C_\kappa(\sigma, \gamma)\zeta^n}{(n+q)!n!} \left(\frac{1-\kappa}{1+\kappa}\right)^{\frac{n+l+n+q+k}{2}} H_{n+l,n+q+k}\left(\frac{\gamma - \kappa\sigma}{\sqrt{1-\kappa^2}}, \frac{\kappa\sigma^* + \gamma^*}{\sqrt{1-\kappa^2}}\right) \right|^2, \tag{39}$$

which is related to a two-variable Hermite polynomial. Especially, when $k = l = 0$, (39) reduces to the Husimi function of PCS.

Next, we plot its three-dimensional graphics in order to see the behavior of the Husimi function of GPAPCS. Figure 6 shows that Husimi function $\mathcal{H}(\sigma, \gamma, \kappa)$ changes against σ

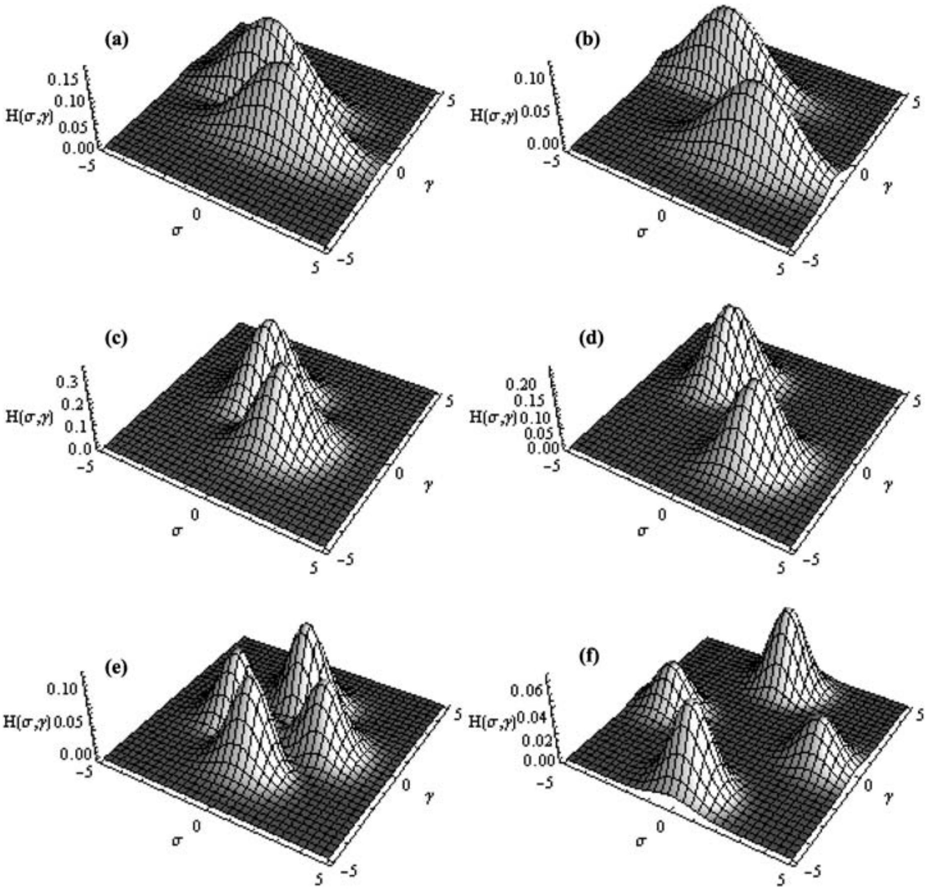


Fig. 6 Husimi functions $\mathcal{H}(\sigma,\gamma)$ of GPAPCS against σ and γ (fixed at $\operatorname{Im}\sigma=\operatorname{Im}\gamma=0$) for $|\zeta|=0.1$: (**a**) $q=1$, $k=l=0$ and $\kappa=0.2$, (**b**) $q=3$, $k=l=0$ and $\kappa=0.2$, (**c**) $q=1$, $k=l=0$ and $\kappa=0.8$, (**d**) $q=1$, $k=2$, $l=0$ and $\kappa=0.8$, (**e**) $q=1$, $k=0$, $l=1$ and $\kappa=0.8$, (**f**) $q=1$, $k=2$, $l=3$ and $\kappa=0.8$

and γ fixed at $\operatorname{Im}\sigma=\operatorname{Im}\gamma=0$ for $|\zeta|=0.1$ with varying the parameters q, k, l and κ. When $k=l=0$, two peaks keep away from each other gradually as the parameter q increases in Fig. 6(a) and (b) and they become more cliffy as the Gaussian parameter κ increases in Fig. 6(a) and (c). The influence of the photon-added numbers q and l is shown in Fig. 6(c)–(f) where $q=1$ and $\kappa=0.8$. From Fig. 6(c) and (d), the distance of two peaks is gradually large with the increase of k. As can be seen in Fig. 6(e) and (f), two peaks change into four peaks for $l\neq 0$. Not only four peaks keep away from each other, but also their maximums become small with the increase of k and l.

5 Conclusions

In summary, we have presented the generalized photon-added pair coherent states $|q,\zeta,k,l\rangle$ and analyzed some prominent nonclassical properties such as sub-Poissonian distribution and violation of Cauchy-Schwarz inequality. We obtain the explicit expression of Wigner function of $|q,\zeta,k,l\rangle$. The result shows that the Wigner function of $|q,\zeta,k,l\rangle$ involves correlation of two two-variable Hermite polynomials, which is not only concise, but also reveals entanglement involved in GPAPCS. Its Husimi function is also calculated, which is also related to a two-variable Hermite polynomial. In addition, we also examine how its Wigner function and the Husimi function varies with the phase space parameters, respectively. It is clear that the nonclassical effects can remarkably be established with increasing the values of q, k and l.

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